Revenue Sharing under Alternative Criteria : A Comparison

1. The Eighth and Ninth Commissions determined the respective shares of States in the devolution of income tax and Union excise duties largely on the basis of three allocative criteria : (i) population (ii) distance, and (iii) inverse of income. While we have used the population and distance criteria, we have not considered it desirable to use inverse of income as a criterion. We have, instead, drawn upon the discussion in paper no. 6 of 1993, NIPFP, New Delhi {Srivastava D.K. and Aggarwal P.K. (1993) "Some Revenue sharing Criteria in Federal Fiscal Systems: Some New Insights"} and developed further the ideas contained therein. Some analytical properties of these criteria are discussed below.

2. The information base for the 'distance' and 'inverse income' criteria consists of the respective pupulations of the State (N_i) and their per capita incomes (y_i). For the population formula, the information base is limited to just (N_i). The subscript i is used here to indicate the i th State. The total number of States is taken to be n. In the ensuing discussion, States have been arranged in an ascending order with respect to per capita income, i.e.

$$y_1 < y_2 < \ldots < y_n \qquad i = 1, 2, \ldots\ldots n$$

3. Shares and per capita shares of States under different criteria have been represented by the following symbols:

Criterion	Share	Per Capita Share
Population	q_i	$q^w_i = q_i / N_i$
Distance	a_i	$a^w_i = a_i / N_i$
Inverse Income	b_i	$b^w_i = b_i / N_i$

The per capita share of a State is derivedby dividing its aggregate share by its population. The following conditions would be satisfied :

$$\Sigma q_i = \Sigma a_i = \Sigma b_i = 1 \qquad i = 1, 2, \ldots\ldots n$$

When the shares are taken as percentages, they would add up to 100 instead of 1.

a. Population Criterion

4. The share of a State in the population formula (q_i) is given by :

$$q_i = N_i / \Sigma N_i$$

The corresponding 'per capita' share is given by

$$q^w_i = 1 / (\Sigma N_i)$$

Since $1 / \Sigma N_i$ (= Q, say) is invariant with respect to Y_i, it means that, in this criterion, the same per capita share is given to each State irrespective of its position on the income scale. In a diagram, where per capita share is indicated on the vertical axis, and per capita income on the horizontal axis, the population based per capita shares would represent a horizontal line (Fig. 1).

b. Distance Criterion

5. In the distance formula, distances are measured by the term ($y_n - y_i$), where y_n is the highest per capita income among all the States. Accordingly, the share of a State in the distance formula may be written as :

$$a_i = N_i (y_n - y_i) / \Sigma N_i (y_n - y_i) \qquad i = 1, 2, \ldots\ldots n$$

The term $1 / \Sigma N_i (y_n - y_i)$ is the same for all the States. Writing this as A, we may rewrite :

$$a_i = A N_i (y_n - y_i)$$

If we divide a_i by N_i, the corresponding per capita share (a^w_i) is obtained. Thus,

$$a^w_i = A (y_n - y_i)$$

6. This equation specifies a straight line which may be represented in a diagram with a^w_i on the y-axis and y_i on the x-axis (Fig.1). This line would fall to the right, since the slope of line ($da^w_i / dy_i = -A$) is negative. It implies that the poorer a State, the larger is its per capita share in the revenue sharing arrangement based on this form of the distance formula. The slope of the line indicates the implied degree of progressivity. It may be noted that the distance formula as written above would given a zero share to the highest income State. Such a version of the formula may be written as its standard or unadjusted version. For a comparision of the relative analytical properties with other allocative criteria, it is a useful starting point. This version of the distance formula has been slightly modified by the last two Finance Commissions, as also by this Commission. The implications of these adjustments have been discussed subsequently.

7. The per capita shares, as determined by the population formula and the distance formula (unadjusted version), may be represented together in one diagram (Fig. 1), with a view to highlighting the implications of bringing progressivity into the allocative scheme. The intersection of the line (a^w_i, q^w_i) is given by :

$$1 / (\Sigma N_i) = A (y_n - y_i)$$

or $$y_i = [y_n - \Sigma N_i (y_n - y_i) / \Sigma N_i]$$

or $$y_i = M$$

Where, M is the average per capita income of all States

$$(= \Sigma N_i y_i / \Sigma N_i)$$

8. This implies that, as compared to the population based shares, States which are below the mean income, get higher shares in the distance formula. Correspondingly, the shares of those States which have per capita incomes higher than the mean income are reduced.

c. Inverse Income Criterion

9. In the inverse income formula, the share of a State may be written as :

$$b_i = (N_i / y_i) / [\Sigma N_i / y_i]$$

Here also, the term $[1 / \Sigma (N_i / y_i]$ is common for all States. Writing this as B, we may rewrite,

$$b_i = B N_i / y_i$$

Dividing this by Ni, we get the corresponding per capita shares (b^W_i). Thus,

$$b^W_i = B/y_i$$

or $(b^W_i)(y_i) = B$

10. This equation describes a rectangular hyperbola in a diagram where $b^W i$ is represented on the vertical axis and yi is represented on the horizontal axis (Fig. 2). In this case also, the line falls to the right as yi increases, indicating progressivity in the revenue sharing arrangement.

11 We may now consider the point of intersection of the q^W_i and b^W_i lines. It is given by :

$$\frac{1}{\sum N_i} = \frac{B}{y_i}$$

or $y_i = \sum N_i / \sum N_i / y_i$

This point will be to the left of mean income

$(M = \sum N_i y_i / \sum N_i)$

if, $M > \sum N_i / \sum N_i / y_i$

or if, $\sum (N_i y_i) [\sum (N_i / y_i] > (\sum N_i)^2$

which is satisfied since the LHS can be written as :

$(\sum N_i)^2$ + interaction terms which are all positive. In other words, the transfer mechanism works in such a way that some of the States that are below average get a share smaller than that assigned to them under the population criterion.

d. Comparison of Distance and Inverse Income Criteria

12. If both a^W_i and b^W_i are brought together in the same diagram (Fig.3), it can be seen that the lines representing per capita shares under the two criteria, i.e. a^W_i and b^W_i, respectively, would intersect at two points. Relative to the distance formula, the inverse income formula favours those States which are very rich or very poor, i.e. States which are located at the two extremes of the income-scale. Conversely, the adjustment that is effected for bringing progressivity into the scheme gives rise to a burden which is borne relatively more by the middle income States in the inverse income formula, as compared to that in the distance formula.

13. The two points of intersection may be identified by using the condition that, for points of intersection, we would have $a^W_i = b^W_i$. Thus,

$$A(y_n - y_i) = B / y_i$$

or $(yi)^2 - (yn)(yi) + B/A = 0$

14. This equation provides the two values of yi (say, u and v) at which the curves representing the per capita shares under the distance and the inverse income formulae intersect. These values are given by:

$$u = .5[y_n - \{(y_n)^2 - 4B/A\}^{1/2}] \text{ and } v = .5[y_n + \{(y_n)^2 - 4B/A\}^{1/2}]$$

15. It can be established that the difference between the per capita shares determined by the distance formula (a^W_i), and the inverse income formula (b^W_i) is maximised when

$$y_i = [\sum (N_i)(y_n - y_i) / \sum (N_i / y_i)]^{1/2}$$

We have, $(a^W_i - b^W_i) = A(y_n - y_i) - B/y_i = z$ (say)

Differentiating the left hand side with respect to y_i, the first order condition for maximisation may be written as:

$$dz/dy_i = -A + B/(y_i)^2$$

This gives $yi = \{B/A\}^{1/2}$

The second order condition for maximisation is also satisfied, since

$$d^2 z / dy_i^2 = -2B / (y_i)^3$$

The ratio $(a^W_i / b^W_i = r)$, (say), on the other hand, is maximised at $y_n/2$, as can be ascertained by writing the relevant first and second order conditions.

16. This indicates that compared to the distance criterion, the inverse income criterion would allocate shares which are relatively higher not only for the poorest State(s) but also the richest State(s) at the cost of the middle income States. The closer the State is to the median income (yn/2), the greater would be its relative loss in the inverse income formula compared to the distance formula.

17. It may be noted that an adjustment has been made in the distance formula used by the Eighth and Ninth Commissions, as also by this Commission, with a view to giving a positive share to the highest income State. The Ninth Commission had used the same notional 'distance' for Goa, Punjab and Maharashtra. This implies that the per capita shares of these States would be equal in the adjusted distance formula. The modification implies that, in the adjusted version of the distance formula, the per capita share of the two richest States would be greater than their corresponding shares in its standard version. This would be reflected in correspondingly reduced shares of the States that are lower on the income scale. These features are indicated in Fig.4.

18. In comparing the per capita shares of States under the distance (standard version), inverse income and population criteria, six points of interest may be identified over the range of income from the lowest per capita income (y_1) to the highest per capita income (y_n). These points are indicated below. The curves representing per capita shares with respect to per capita income under the alternative criteria have been referred to as the distance, inverse-income and population criteria curves, respectively.

(i) u : the point of intersection between the distance curve (a^W_i) and inverse income curve (b^W_i) at the lower end of per capita incomes ;

(ii) v : point of intersection between the two curves, at the higher end of per capita incomes ;

(iii) M : the mean income defined by $\sum N_i y_i / \sum N_i$. This gives the point of intersection of the population criterion curve (q^W_i) with the distance curve (a^W_i).

(iv) y(q,b) : This is given by $(\sum N_i / \sum N_i / y_i)$. This gives the point of intersection of the population criterion curve (q^W_i) with the inverse income curve.

(v) $\{B/A\}^{1/2}$: This is the point at which the difference between the per capita shares determined by the distance formula and the inverse income formula, i.e. $(a^W_i - b^W_i)$ is maximised.

(vi) yn/2 : This is the point at which the ratio between the per capita shares under the distance and the inverse income formulae (a^W_i / b^W_i) is maximised.

19. The income-levels corresponding to the six points mentioned above have been calculated with respect to a distribution of (y_i, N_i), where y_i refers to the per capita incomes of States calculated as an average of per capita incomes of 1987-88, 1988-89 and 1989-90, and population figures relate to the 1971 census. In Table 1, the States have been arranged according to an ascending order of per capita income. The critical income levels corresponding to the six points identified earlier are given in this Table.

20. Between the distance formula and the inverse income formula, the use of the latter would benefit Bihar at the lower end and the States from Arunachal Pradesh to Goa at the upper end of the income scale (Table 1). The difference between the two is maximised at about the income levels of Jammu & Kashmir and Himachal Pradesh. The intersection between the population and inverse income curves takes place at an income level just below that of Meghalaya. Between this and the mean income level, there are five States, viz. Himachal Pradesh, Jammu and Kashmir, Kerala, Andhra Pradesh and Manipur.

21. In Table 2, the shares of States determined under the three formulae, viz. population, distance and inverse income formulae have been given using the distribution of Ni based on 1971 population and per capita incomes (y_i) that represent the average of three years, viz. 1987-88, 1988-89 and 1989-90. The corresponding per capita shares are given in Table 3.

22. A comparison of the per capita shares under the alternative version of the distance criterion indicates that, as compared to the standard version, the adjusted distance formula allocates higher shares to Goa and Punjab at the upper end of the income-scale, and Rajasthan, Orissa, Uttar Pradesh and Bihar at the lower end of the income scale.

Per Capita Shares Under Alternative Criteria

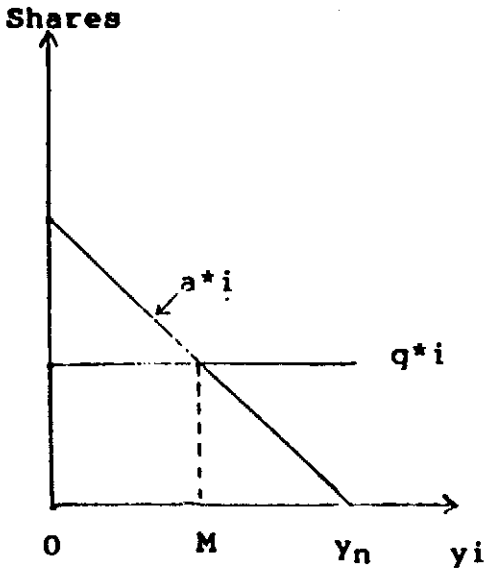

Fig. 1

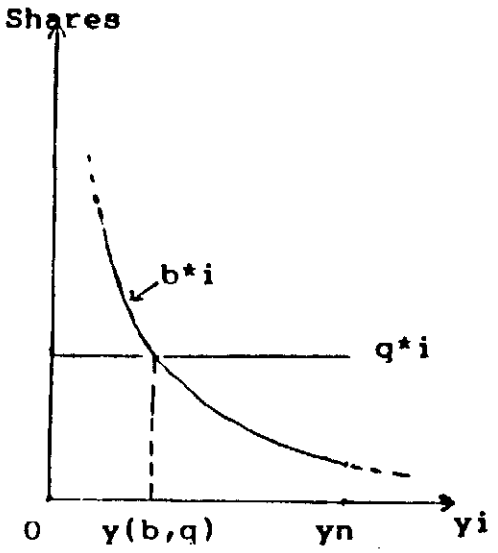

Fig. 2

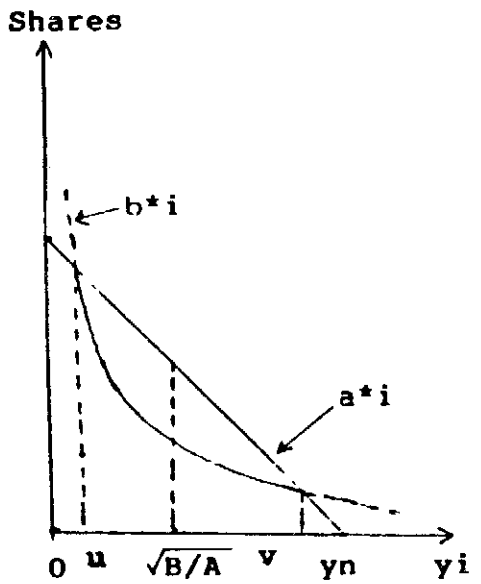

Fig. 3

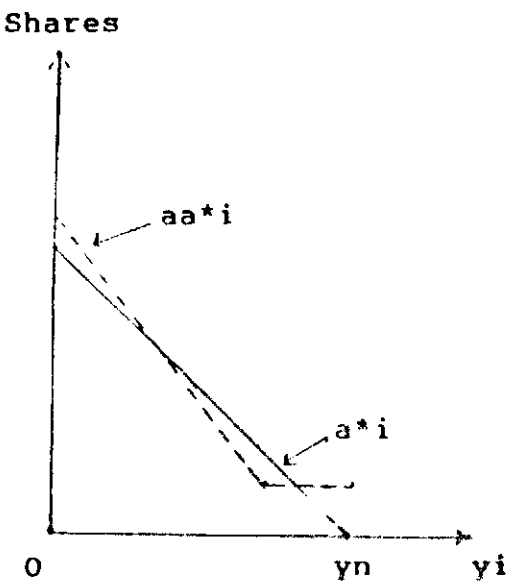

Fig. 4

Shares refer to per capita shares
yi indicates per capita income.
aa*i refers to per capita shares under the adjusted distance formula.

Table 1

Per Capita Incomes : Points of Interest under Alternative Criteria

State	Per Capita Income (Rs.)	Critical income levels (Rs.)	Intersection between curves
Bihar	2135		
		u =2699	distance & inverse income
Uttar Pradesh	2867		
Orissa	2945		
Rajasthan	3092		
Tripura	3163		
Assam	3195		
Madhya Pradesh	3299		
Meghalaya	3328		
		y(q,b)=3358	population & inverse income
Manipur	3449		
Andhra Pradesh	3455		
Kerala	3532		
Jammu & Kashmir	3534		
		$\{B/A\}^{1/2}$=3548	*
Himachal Pradesh	3618		
		M = 3625	population & distance
		$y_n/2$ = 3682	**
West Bengal	3750		
Karnataka	3810		
Nagaland	3929		
Tamil Nadu	4093		
Mizoram	4094		
Gujarat	4602		
		v =4665	distance & inverse income
Arunachal Pradesh	4670		
Sikkim	4846		
Haryana	5284		
Maharashtra	5369		
Punjab	6996		
Goa	7364		

* *Income level at which the difference between per capita shares under distance and inverse income criteria ($a^W i$-$b^W i$) is maximised.*

** *Income level at which the ratio $a^W i/b^W i$ is maximised.*

Table 2

Alternative Criteria : State-wise Shares

States arranged in ascending order of income	Average (1987-90) Per Capita Income (Rupees)	Population (in lakhs) 1971 Census	Shares Under Alternative criteria (Per cent) Population	Distance	Inverse Income	Adjusted Distance
1	2	3	4	5	6	7
Bihar	2135	563.53	10.377	14.513	16.367	14.773
Uttar Pradesh	2867	883.41	16.267	19.566	19.107	19.672
Orissa	2945	219.45	4.041	4.776	4.621	4.795
Rajasthan	3092	257.66	4.744	5.421	5.167	5.425
Tripura	3163	15.56	0.287	0.322	0.305	0.322
Assam	3195	146.25	2.693	3.003	2.839	2.998
Madhya Pradesh	3299	416.54	7.670	8.339	7.830	8.305
Meghalaya	3328	10.12	0.186	0.201	0.189	0.200
Manipur	3449	10.73	0.198	0.207	0.193	0.205
Andhra Pradesh	3455	435.03	8.010	8.375	7.808	8.308
Kerala	3532	213.47	3.931	4.029	3.748	3.988
Jammu & Kashmir	3534	46.17	0.850	0.871	0.810	0.862
Himachal Pradesh	3618	34.60	0.637	0.638	0.593	0.630
West Bengal	3750	443.12	8.159	7.887	7.327	7.757
Karnataka	3810	292.99	5.395	5.128	4.769	5.034
Nagaland	3929	5.16	0.095	0.087	0.081	0.085
Tamil Nadu	4093	411.99	7.586	6.637	6.242	6.450
Mizoram	4094	3.32	0.061	0.053	0.050	0.052
Gujarat	4602	266.97	4.916	3.632	3.597	3.447
Arunachal Pradesh	4670	4.68	0.086	0.062	0.062	0.059
Sikkim	4846	2.10	0.039	0.026	0.027	0.024
Haryana	5284	100.37	1.848	1.028	1.178	0.927
Maharashtra	5369	504.12	9.283	4.953	5.822	4.423
Punjab	6996	135.51	2.495	0.246	1.201	1.189
Goa	7364	7.95	0.146	0.000	0.067	0.070
		5430.80	**100.000**	**100.000**	**100.000**	**100.000**

Table 3

Alternative Criteria : Per Capita Shares

States arranged in ascending order of income	Per Capita Shares x 10,000 (based on 1971 population)			
	qo*	ao*	bo*	aao*
1	2	3	4	5
Bihar	184.14	257.53	290.44	262.16
Uttar Pradesh	184.14	221.47	216.29	222.68
Orissa	184.14	217.63	210.56	218.47
Rajasthan	184.14	210.39	200.55	210.54
Tripura	184.14	206.89	196.05	206.72
Assam	184.14	205.32	194.08	204.99
Madhya Pradesh	184.14	200.20	187.97	199.38
Meghalaya	184.14	198.77	186.33	197.82
Manipur	184.14	192.81	179.79	191.29
Andhra Pradesh	184.14	192.52	179.48	190.97
Kerala	184.14	188.73	175.57	186.82
Jammu & Kashmir	184.14	188.63	175.47	186.71
Himachal Pradesh	184.14	184.49	171.39	182.18
West Bengal	184.14	177.99	165.36	175.06
Karnataka	184.14	175.03	162.76	171.82
Nagaland	184.14	169.17	157.83	165.40
Tamil Nadu	184.14	161.10	151.50	156.56
Mizoram	184.14	161.05	151.47	156.51
Gujarat	184.14	136.03	134.75	129.11
Arunachal Pradesh	184.14	132.68	132.78	125.44
Sikkim	184.14	124.01	127.96	115.95
Haryana	184.14	102.44	117.35	92.33
Maharashtra	184.14	98.25	115.50	87.74
Punjab	184.14	18.12	88.64	87.74
Goa	184.14	0.00	84.21	87.74

Per Capita shares under different formulae have been indicated as detailed below:

qo = population criterion;*

ao = distance criterion (standard version);*

bo = inverse-income criterion;*

aao = adjusted distance criterion.*